DYNAMIC BUCKLING OF SHALLOW SPHERICAL CAPS SUBJECTED TO A NEARLY AXISYMMETRIC STEP PRESSURE LOAD

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THESIS

DYNAMIC BUCKLING OF SHALLOW SPHERICAL CAPS
SUBJECTED TO A NEARLY AXISYMMETRIC
STEP PRESSURE LOAD

by

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Dynamic Buckling of Shallow Spherical Caps Subjected to a Nearly Axisymmetric Step Pressure Load

by

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ABSTRACT

The behavior of shallow spherical caps subjected to axisymmetric and nearly axisymmetric step pressure loads is examined using a digital computer program for the geometrically nonlinear analysis of arbitrarily loaded shells of revolution. A criterion for dynamic buckling under the nearly axisymmetric load is proposed and critical buckling pressures are determined for both loading conditions for a large range of cap sizes. The axisymmetric results either compare well with, or are somewhat higher than, previously published theoretical results. For most of the cap sizes the buckling pressures from the asymmetric analysis are new results. A numerical convergence study is conducted to verify the accuracy of the results.

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TABLE OF SYMBOLS

E	the modulus of elasticity of the shell
H	the rise of the spherical cap at the pole
h	the thickness of the shell
$K_L^{(n)}$	the square matrix associated with the linear portion of the governing equations
m	the mass density of the shell
M_s	the meridional bending moment per unit length
n	the Fourier index
P	a nondimensional applied load
P_{CRIT}	the nondimensional critical pressure
q_o	the classical buckling pressure of a complete sphere
$q^{(n)}$	a column matrix containing the coefficients of the n^{th} term in the series expansion of the applied load
$q_{NL}^{(n)}$	a column matrix containing the inertial and nonlinear terms in the n^{th} set of equations
r	the normal distance from the axis of revolution to the surface of the cap
r_o	the normal distance from the axis to the cap in the base plane; the maximum value of r
R_s, R_θ	the radii of curvature in the s and θ directions respectively
s	the meridional distance along the surface of the shell
t	the nondimensional time
T	the time
T_o	a reference time
U, V, W	the displacements in the s, θ and ζ directions respectively

$\bar{V}$	a nondimensional measure of the volume of the shell deformation
$\bar{V}_{MAX}$	the peak in the time history of the parameter $\bar{V}$
$W^{(n)}$	the displacement in the ζ direction in the nth harmonic
δt	the nondimensional time increment
$\varepsilon^{(n)}$	the nondimensional parameter governing the magnitude of the load applied in the asymmetric harmonics
ζ	the coordinate normal to the surface of the shell
θ	the circumferential angle measured about the axis of revolution
λ	a nondimensional geometric parameter used to describe the spherical cap
ν	Poisson's ratio
ξ	the normal distance from the base plane to the middle surface of the undeformed cap

I. INTRODUCTION

The critical pressures for the static axisymmetric snap buckling of uniformly loaded clamped shallow spherical caps were presented by Huang [1] in 1964. Results for the critical pressures for dynamic axisymmetric buckling of clamped shallow spherical caps subjected to a step pressure load have been recently given by Huang [2], by Stephens and Fulton [3], by Stricklin et al. [4] and by Ball [5]. The results for the axisymmetric dynamic buckling indicate that the critical dynamic pressure varies from 40% to 75% of the critical pressure for static axisymmetric buckling, depending upon the size of the cap. In his static analysis, Huang [1] shows that asymmetric buckling modes exist at pressures below the critical pressure for static axisymmetric snap buckling. These asymmetric modes are unstable and cause the cap to buckle at the minimum bifurcation buckling pressure. If imperfections are present the buckling will occur at pressures below the minimum bifurcation buckling pressure.

This thesis considers the possibility that the same situation may exist for the dynamically loaded cap, i.e., the presence of asymmetric motion may reduce the critical dynamic buckling pressure. A digital computer program for the geometrically nonlinear analysis of arbitrarily loaded shells of revolution [5] is used to determine the response

of several caps when subjected to axisymmetric and nearly axisymmetric step pressure loads. It was anticipated that the presence of the very small asymmetric portion of the load would initiate asymmetric motion and that this motion might grow large and significantly influence the axisymmetric response. If the axisymmetric response is not influenced by the asymmetric motion, the buckling is referred to as direct snapping; if the asymmetric motion reduces the critical pressure, the buckling is called indirect snapping [6]. Stricklin et al. [4] have also considered the dynamic asymmetric buckling problem and have presented a range of critical pressures for three cap sizes. The objective of the thesis is to determine more precisely the critical dynamic pressure for a large range of cap sizes.

II. DESCRIPTION OF THE COMPUTER PROGRAM

The computer program used in this study is capable of analyzing thin, linearly elastic shells of revolution subjected to arbitrary load and temperature distributions. The governing partial differential equations are based upon Sanders' nonlinear thin shell theory for the condition of small strains and moderately small rotations [7]. The inplane and normal inertial forces are accounted for, but the rotary inertial terms are neglected. The set of governing nonlinear partial differential equations is reduced to an infinite number of sets of four second-order differential equations in the meridional and time coordinates by expanding all dependent variables in a sine or cosine series in terms of the circumferential coordinate θ. The sets are uncoupled by using appropriate trigonometric identities and by treating the nonlinear coupling terms as pseudo loads. The meridional derivatives are replaced by the conventional central finite difference approximations, and the displacement accelerations are approximated by the implicit Houbolt backward differencing scheme [8].

This leads to sets of algebraic equations in terms of the dependent variables and the Fourier index, n, of the form

$$[K_L^{(n)}] \{Z^{(n)}\} = \{q^{(n)}\} + \{q_{NL}^{(n)}\} \qquad (1)$$

The square matrix $[K_L^{(n)}]$ is a banded matrix associated with the linear portion of the governing equations. The elements in the column matrix $\{Z^{(n)}\}$ are the coefficients of the n^{th} term in the series expansions of U, V, W, and M_s at each finite difference station where U, V. and W are the displacements in the meridional, circumferential and normal directions respectively, and M_s is the meridional bending moment per unit length. The applied loads in the n^{th} harmonic are contained in the column matrix $\{q^{(n)}\}$, and the inertial and nonlinear terms are contained in the column matrix $\{q_{NL}^{(n)}\}$.

At each time step, an estimate of the solution is obtained by extrapolation from the solutions at the previous time steps. An elimination procedure is used to repeatedly solve each set of algebraic equations until the maximum difference between two consecutive solutions $\{Z^{(n)}\}$, $n = 0,1,2,\ldots$, is less than the convergence criterion, which is a specified percentage of the maximum value of U, V, W, and M_s in the n^{th} harmonic. A time history of the shell response is obtained by incrementing the time in equal steps until either a prescribed maximum number of time steps have been taken or the solution fails to converge after a specified number of iterations. The time interval is usually so small only a few iterations are required since the difference between the estimated solution and computed solution is generally small. However, when the shell becomes dynamically unstable, the solution may not converge, even with many iterations.

The program is dimensioned such that the product of the number of meridional stations and the number of harmonics must be less than 201, and the maximum number of harmonics must be less than 11. The program occupies approximately 155,000 bytes on the IBM 360/67 Digital Computer, and a typical execution time using 40 stations and 2 harmonics with 3000 time steps was 12 minutes using the FORTRAN IV, LEVEL H Compiler, with OPT = 2. A copy of the program for a static analysis and USERS MANUAL can be obtained through COSMIC (M70-10098,LAR-10736). The program with the dynamic analysis capability will be available in the latter half of 1971.

III. PROBLEM DESCRIPTION

The geometry of the shallow spherical cap can be specified by the nondimensional parameter λ, where

$$\lambda = 2[3(1-\nu^2)]^{\frac{1}{4}}(H/h)^{\frac{1}{2}} \tag{2}$$

and H is the rise of the cap, h is the thickness and ν is Poisson's ratio. The mass density of the cap is m. For all shells considered here:

Radii of curvature, R_s, R_θ = 250 in.
Thickness, h = 0.25 in.
Modules of Elasticity, E = 30×10^6 psi
Poisson's ratio, ν = 0.3

The classical buckling pressure of a complete sphere q_o is given by

$$q_o = \frac{2E}{\sqrt{3(1-\nu^2)}}(h/R_s)^2 \tag{3}$$

The nearly axisymmetric step pressure load is given by

$$\begin{aligned} \{q^{(0)}\} &= Pq_o\{1\} \\ \{q^{(n)}\} &= Pq_o\varepsilon^{(n)}\{1\},\ n > 0,\ T \geq 0 \end{aligned} \tag{4}$$

in which P is a nondimensional load parameter, $\varepsilon^{(n)}$ is a small number and T is time. The shell is stress free and motionless prior to the application of the load. For the axisymmetric analysis, only the n = 0 harmonic is considered.

In the asymmetric analysis various harmonics were excited either singly or in groups through the parameter $\varepsilon^{(n)}=0.0001$.

The nondimensional time is denoted by t, where

$$t = T/T_o \tag{5}$$

and

$$T_o = (R_s^2 m/E)^{\frac{1}{2}} \tag{6}$$

For this study, m was selected such that T_o was equal to one. For most of the runs, the non-dimensional time increment δt was taken as 0.05 and the maximum response time was 150. The necessity for the long response time was noted in Reference 3.

IV. BUCKLING CRITERIA

Criteria for dynamic buckling of shells are not as well defined as they have been for the case of static buckling. Quoting from Stoker [9], ". . . stability in dynamics in an idea that means different things to different people, . . ." and "Criteria for stability that are logically sound, and also practically applicable, are not easy to come by either."

A. AXISYMMETRIC ANALYSIS

The criterion most often used for dynamic axisymmetric buckling of the spherical cap is based on the variation of $\overline{V}_{MAX}$, the peak in the time history of a nondimensional average displacement parameter $\overline{V}$, with respect to P, the amplitude of the applied load [2, 3]. The parameter $\overline{V}$ is a measure of the volume of the shell deformation and has been defined in Reference 3 as

$$\overline{V} = \frac{\int_0^{r_o} rW^{(0)} \, dr}{\int_0^{r_o} r \, \xi \, dr} \qquad (7)$$

in which r is the normal distance from the axis to the cap, r_o is the maximum value of r, and ξ is the vertical distance from the base plane to the undeformed cap middle surface. For loads below the dynamic buckling load, where the nonlinearities are small, the relationship between $\overline{V}$ and P is

approximately linear. There may exist, however, a certain value of P at which a very small increase in P produces a very large increase in $\overline{V}_{MAX}$. This load is defined in References 2 and 3 as the buckling load.

B. ASYMMETRIC ANALYSIS

At the present time no criterion for the dynamic buckling of shallow spherical caps under nearly axisymmetric loading conditions has been proposed. The results of this study suggest that the criterion for buckling in the asymmetric analysis is identical to the criterion for buckling in the axisymmetric analysis, i.e., buckling is defined to occur when a small change in the nearly axisymmetric applied load causes a large change in $\overline{V}_{MAX}$, where $\overline{V}$ is given by equation (7). This criterion is based upon the argument that the axisymmetric deformation is the only true measure of buckling. The cap can suffer large displacements in the asymmetric harmonics, but these displacements in themselves do not represent a catastrophic buckling failure because the transverse displacements at the pole and base of the cap are zero in the asymmetric harmonics. Thus, the shell will have snapped through, or buckled, only when $\overline{V}$, the volume of deformation, becomes large. The onset of buckling may occur at loads smaller than those that cause buckling in the axisymmetric analysis because of the influence of motion in the asymmetric harmonics on the axisymmetric response.

For convenience, the response in each asymmetric harmonic will be measured using equation (7) with $W^{(0)}$ replaced with $W^{(n)}$. The parameter $\overline{V}$ for the asymmetric harmonics does not represent a volume of deformation as it does for the axisymmetric harmonic. It can, however, be used to indicate the relative excitation of the asymmetric harmonics.

V. RESULTS

A. AXISYMMETRIC ANALYSIS

Plots of $\overline{V}$ versus t for two values of P are given in Figures 1 through 3 for $\lambda = 5$, 6 and 9 respectively. In Figures 1 and 3 one value of P is slightly above the critical pressure. In Figure 2 both values of P are slightly above the critical pressure. Note that the shells $\lambda = 5$ and 6 buckle considerably earlier than the shell $\lambda = 9$. The shells $\lambda = 8$ and 10 also buckled relatively early, while the shells $\lambda = 7$, 7.5, 11, 12 and 13 responded in a manner similar to the shell $\lambda = 9$ and oscillated many cycles before buckling. When the applied pressure was somewhat larger than the critical pressure, the time to buckle for most of the shells decreased, as illustrated in Figure 2. However, the shell $\lambda = 13$ took longer to buckle for a pressure approximately 10% larger than the critical pressure.

A plot of $\overline{V}_{MAX}$ versus P is given in Figure 4 for $\lambda = 4$, 5,..., 13 and 7.5.[1] Note the distinct break in the relationship between P and $\overline{V}_{MAX}$ for all λ except $\lambda = 4$.[2] In every case the point after the break was accompanied by a

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[1] The results for the shells $\lambda = 5$, 8, and 11 were first presented in Reference 5.

[2] The shell $\lambda = 4$ did not exhibit a distinct break in the P - $\overline{V}_{MAX}$ relationship. Hence, there is no well defined critical pressure. Nevertheless, there is a large change in $\overline{V}_{MAX}$ over a relatively small range of pressure. A similar situation exists for the static buckling of shells with small λ.

convergence failure. Hence, larger values of $\overline{V}_{MAX}$ would have occurred if the program could have continued. The value of P at the break is defined as the critical pressure, P_{CRIT}, and is given in Table I and in Figure 5 for the values of λ considered. Figure 5 also shows the results presented by Huang [2], by Stephens and Fulton [3] and by Stricklin et al. [4]. The results for static axisymmetric buckling presented by Huang [1] and by Stilwell [10], who used the same program as the present study, are also shown in Figure 5 for purposes of comparison.

B. ASYMMETRIC ANALYSIS

Due to the significant increase in computer time required to examine several harmonics simultaneously, only a few of the harmonics in the vicinity of the critical harmonic for static buckling were excited. To verify the validity of this procedure three shells were examined using additional harmonics. These results will be presented in another section.

As an example of a typical response curve, a plot of both $W^{(n)}/H$ and $\overline{V}$ at $r = 0.513\ r_o$ for the harmonics $n = 0$ and 2 is given in Figure 6 for the shell $\lambda = 6$ with $P = 0.566$. The critical harmonic for static buckling of this shell is $n = 2$. Note in Figure 6 the agreement between $W^{(n)}/H$ and $\overline{V}$ in both harmonics. Also shown in Figure 6 is $\overline{V}$ for $n = 0$ and $P = 0.566$ without the presence of the asymmetric harmonics.

Similar runs were made for = 5, 6, 7, ..., 13 and 7.5 with several values of P. Only the shells $\lambda = 6, 7, 7.5$, 11 and 12 were influenced by the asymmetric motion. The other shells were not influenced by the motion in the asymmetric harmonics and buckled at the critical pressure from the axisymmetric analysis. The plots of $\overline{V}_{MAX}$ for $n = 0$ versus P are shown in Figure 7 for the five shells. Note the distinct break in the $\overline{V}_{MAX}$ - P relationship. Furthermore, the point after the break was accompanied by a convergence failure. However, the point at P = .52 for the shell $\lambda = 6$ was a converged solution.

The critical pressures for all the shells examined in the asymmetric analysis are given in Table II and in Figure 8. The range of critical pressures reported by Stricklin et al. [4] are shown in Figure 8 by bars encompassing the span of values for which they found appreciable excitation of asymmetrical displacement coefficients. The experimental results for two shells with $\lambda = 7.5$ presented by Lock et al. [6] are also given in Figure 8.

VI. DISCUSSION

The major results of this study are the critical pressures from the asymmetric analysis given in Figure 8. These results show that when the critical pressure from the axisymmetric analysis is larger than approximately 0.52, the shell will buckle at $P \simeq 0.52$ due to the presence of motion in the critical asymmetric harmonic. In this analysis, the motion in each asymmetric harmonic was caused by a load of .01% of the step pressure load. An analysis that uses very small asymmetric imperfections in the shape of the cap as the triggering mechanism should give the same results.

The two experimental buckling pressures presented in Reference 6 for the shell $\lambda = 7.5$ are below the value predicted in this study. The discrepancy is probably due to the imperfection sensitivity of the spherical cap. According to Reference 6, the response of the two shell models was significantly different. One shell buckled shortly after the load was applied; the other shell vibrated for about nine cycles prior to snapping. The reason for this can be seen in Figure 8. The shell $\lambda = 7.5$ is almost a borderline case and the buckling may or may not be influenced by asymmetric motion, depending upon the exact properties of the shell. In Reference 6, direct snapping was characterized by a rapid growth in the axisymmetric response

with little oscillation. An example of this type of behavior is shown in Figure 1. On the other hand, Figure 3 with P = 0.44 illustrates a different kind of direct snapping behavior since many oscillations occur prior to buckling. However, an additional increase in the load caused most of the shells to buckle much sooner as illustrated in Figure 2. The indirect snapping discussed in Reference 6 was characterized by several oscillations in the axisymmetric harmonic with a steady growth in an asymmetric harmonic until snapping occurred in the axisymmetric mode. The response shown in Figure 6 is an illustration of this type of behavior. Thus, a shell may oscillate several cycles before buckling if either the load is equal to or slightly above the critical load or the shell buckles due to asymmetric motion. Hence, the shell $\lambda = 7.5$ could exhibit both direct snapping and indirect snapping.

VII. NUMERICAL CONVERGENCE STUDY

The numerical parameters used in the analysis could, if not selected properly, lead to inaccurate solutions. Consequently, the influence of the time step, the convergence criterion, the number of finite difference stations along the meridian, double precision versus single precision, and the assumed critical dynamic asymmetric harmonic was examined for several values of λ.

The time step was reduced from 0.05 to 0.025 for the axisymmetric analysis of the shells $\lambda = 5$ and 11. In References 2 and 3, $\delta t = 0.02$ and 0.10 respectively. For the shell $\lambda = 5$, the critical load was 0.48, as before; but the solution at $P = 0.49$, which displayed a sharp rise in $\overline{V}_{MAX}$, did not experience a convergence failure. The smaller time step allowed the solution to enter the post-buckling region. For the shell $\lambda = 11$, $P_{CRIT} = 0.575$, and the solution failed to converge at $P = 0.58$, as before.

The results presented in Figures 1 - 8 were obtained using a convergence criterion of .01. To see if this was adequate, the critical axisymmetric pressure of the shell $\lambda = 10$ was determined using several smaller values. Decreasing the value of the criterion to 0.001 and to 0.0001 resulted only in requiring more iterations (a maximum of two were required at 0.01, three were required at 0.001 and four were required at 0.0001) with no change in the value

of $\overline{V}_{MAX}$ and P_{CRIT}. The computer execution time increased from approximately six minutes for 0.01 to approximately twelve minutes for 0.001 and eighteen minutes for 0.0001.

The number of difference stations used to model the shell was forty. This agreed with number found to be sufficient for the static analysis for $\lambda \leq 10$ [10]. Twenty-six stations were used in Reference 3 and Reference 2 reported using $4\lambda + 1$ stations. The number of stations was increased to eighty for the axisymmetric analysis of the shell $\lambda = 11$. No significant change was found in the results.

The computations for all the shells were made using the single precision mode of the IBM 360/67. Results for the shells $\lambda = 6$ and 10 were also obtained using the double precision mode. The double precision arithmetic produced no appreciable change in the critical axisymmetric pressures.

In order to establish the validity of the assumption that the critical asymmetric harmonic is in the vicinity of the critical harmonic for static buckling, the solution for the shell $\lambda = 6$ was obtained using the four harmonics $n = 0, 1, 2$ and 3. The critical harmonic for static buckling is $n = 2$. The results showed that the response in the harmonics $n = 1$ and 3 was relatively insignificant, but, as shown in Figure 6, $\overline{V}$ for the harmonic $n = 2$ was of the same order of magnitude as $\overline{V}$ for the axisymmetric harmonic. Hence, the critical asymmetric static harmonic appears to also be the critical asymmetric dynamic harmonic. A check

on the critical asymmetric harmonic was also made for the shell $\lambda = 12$. The critical static harmonic is $n = 7$. Combinations of harmonics from $n = 4$ to $n = 9$ were excited. Although there was no appreciable change in the critical pressure, harmonics $n = 5$ and 6, not 7, showed the largest motion. The shell $\lambda = 9$ with the critical asymmetric static harmonics $n = 4$ and 5, was examined using combinations of harmonic $n = 1$ to $n = 5$. None of these harmonics was appreciable in comparison with the axisymmetric motion, nor did they have any effect on the critical pressure.

VIII. SUMMARY AND CONCLUSIONS

A digital computer program for the geometrically nonlinear analysis of arbitrarily loaded shells of revolution has been used to determine the minimum buckling load of clamped shallow spherical caps subjected to axisymmetric and nearly axisymmetric step pressure loads. For both loading conditions, the critical pressure was defined to be that pressure which when given a small increase caused a large change in the volume of deformation. In the axisymmetric analysis, the critical pressure was determined for the shells $\lambda = 5$, 6,..., 13 and 7.5. In general, the results either agree well or are somewhat higher than previously published results. In the asymmetric analysis, motion was introduced in several of the harmonics in the vicinity of the critical harmonic for static buckling. Those shells with a critical pressure from the axisymmetric analysis greater than about 52% of the classical buckling pressure of a complete sphere ($\lambda = 6$, 7, 7.5, 11 and 12) buckled at a pressure ratio of approximately 0.52 due to the asymmetric motion. The other shells ($\lambda = 5$, 8, 9, 10 and 13) buckled at pressure ratios below 0.52 and were not affected by the asymmetric motion.

These results were obtained using a perfect shell of revolution with a very small load in each asymmetric harmonic. The critical pressures of actual imperfect shells

will be less than the pressures presented here. How much less depends upon the nature of the initial imperfections and is a subject for future study.

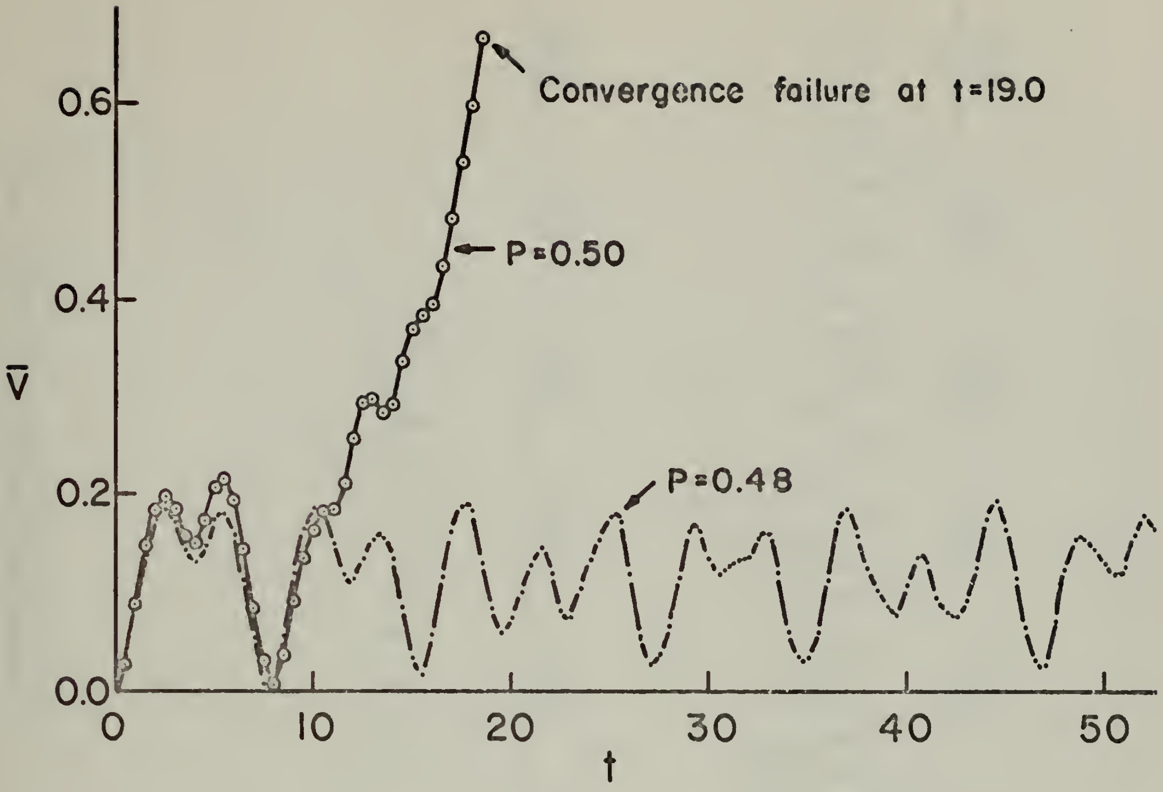

Figure 1. Time History of $\bar{V}$ for two values of P, $\lambda = 5$.

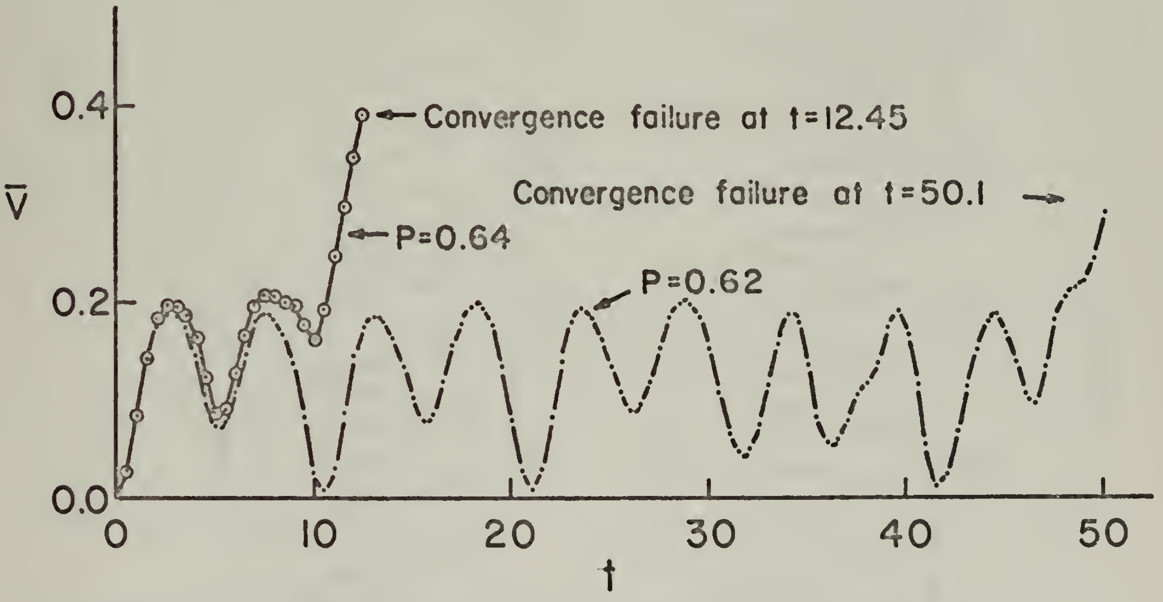

Figure 2. Time History of $\bar{V}$ for two values of P, $\lambda = 6$.

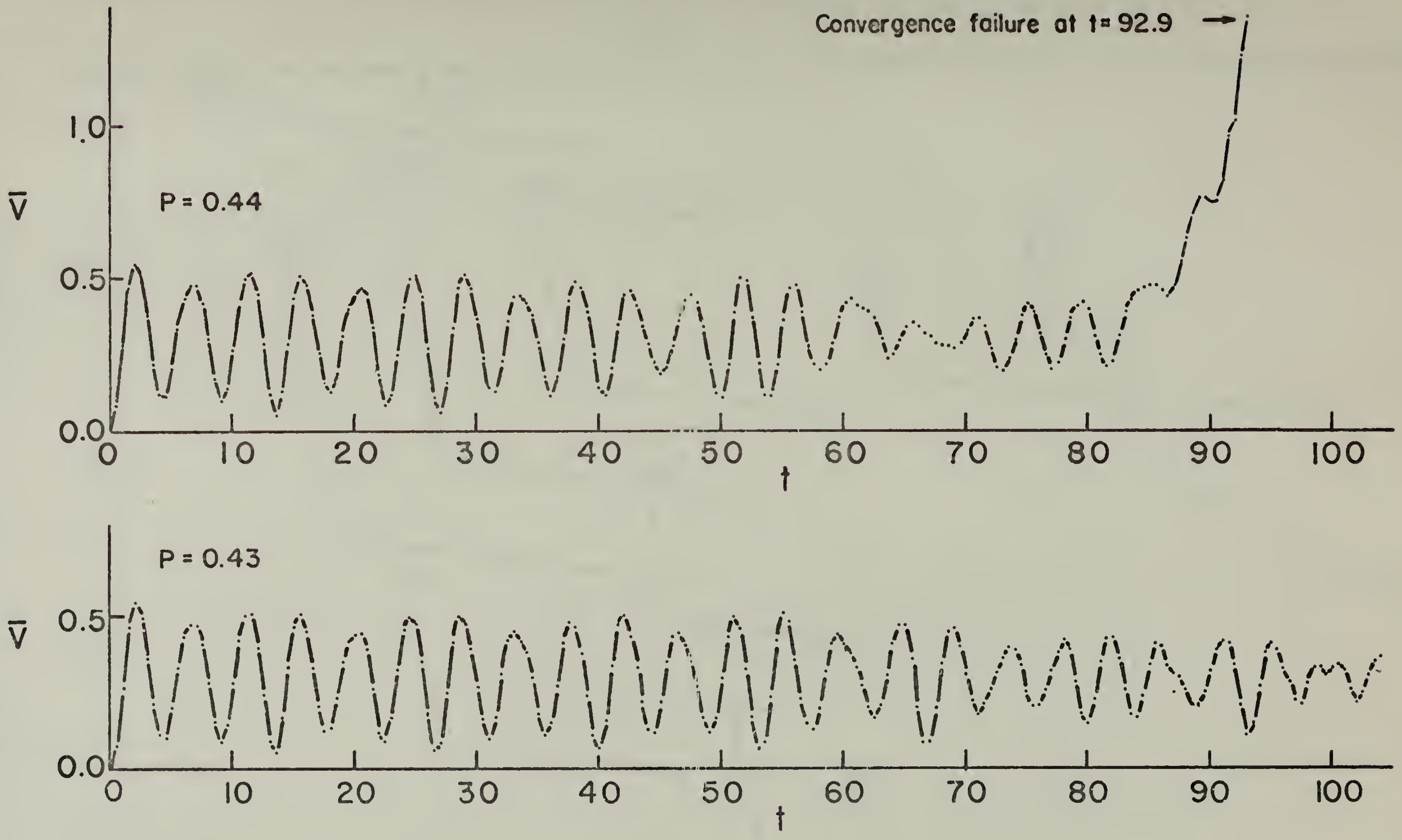

Figure 3. Time History of $\overline{V}$ for two values of P, λ = 9.

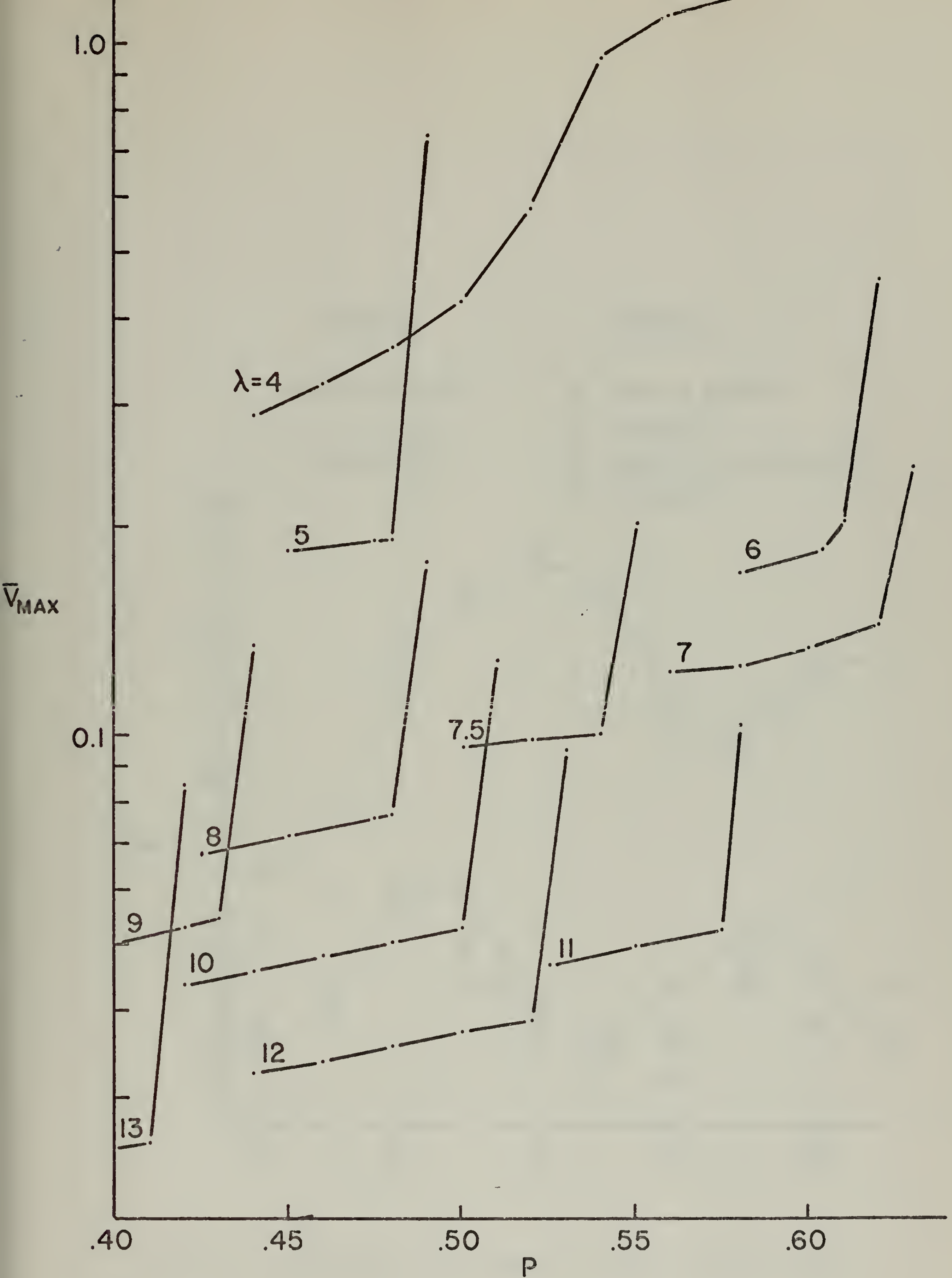

Figure 4. Peak Deflection versus step pressure load from axisymmetric analysis for various values of λ.

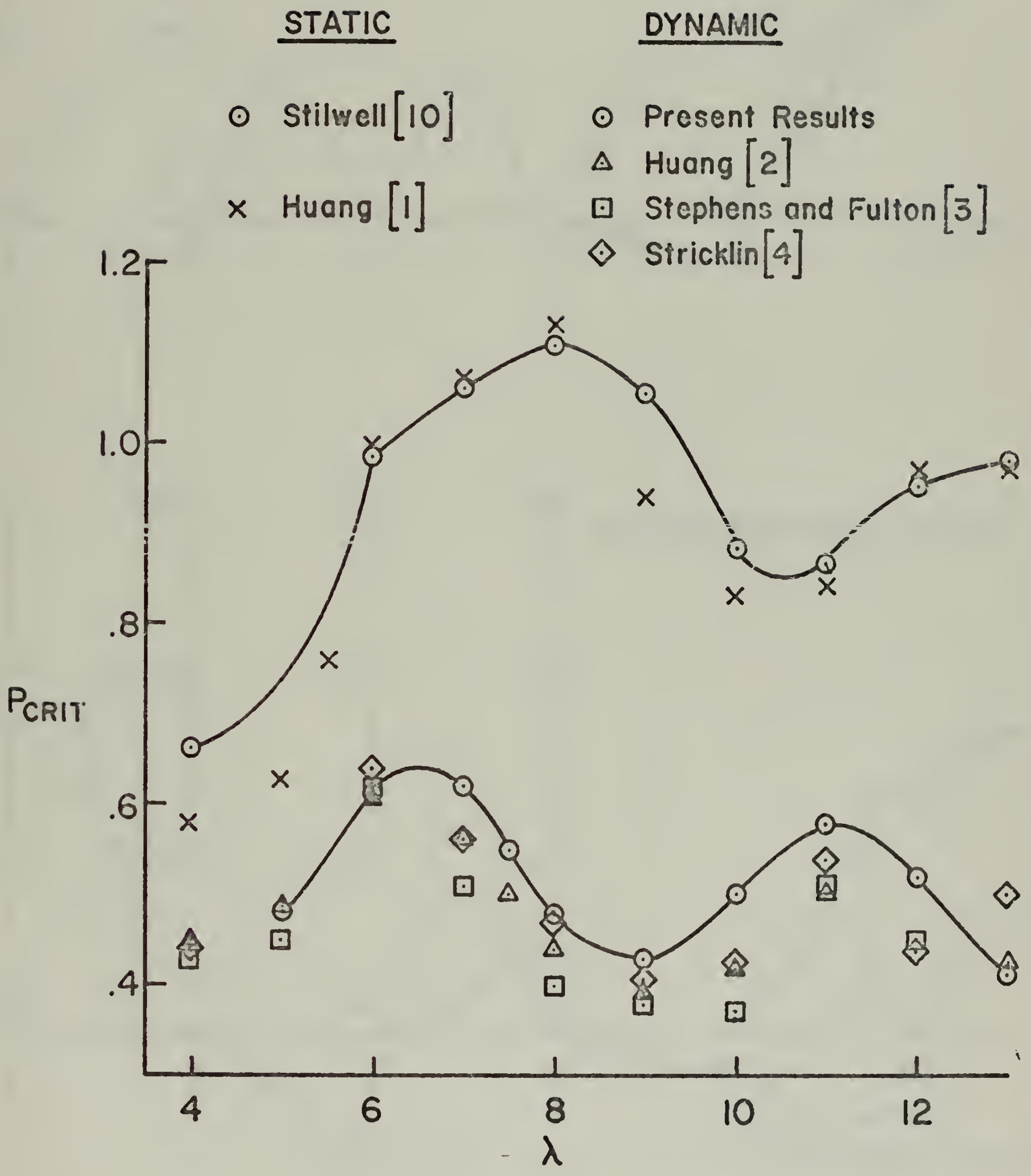

Figure 5. Critical axisymmetric step pressure load versus λ.

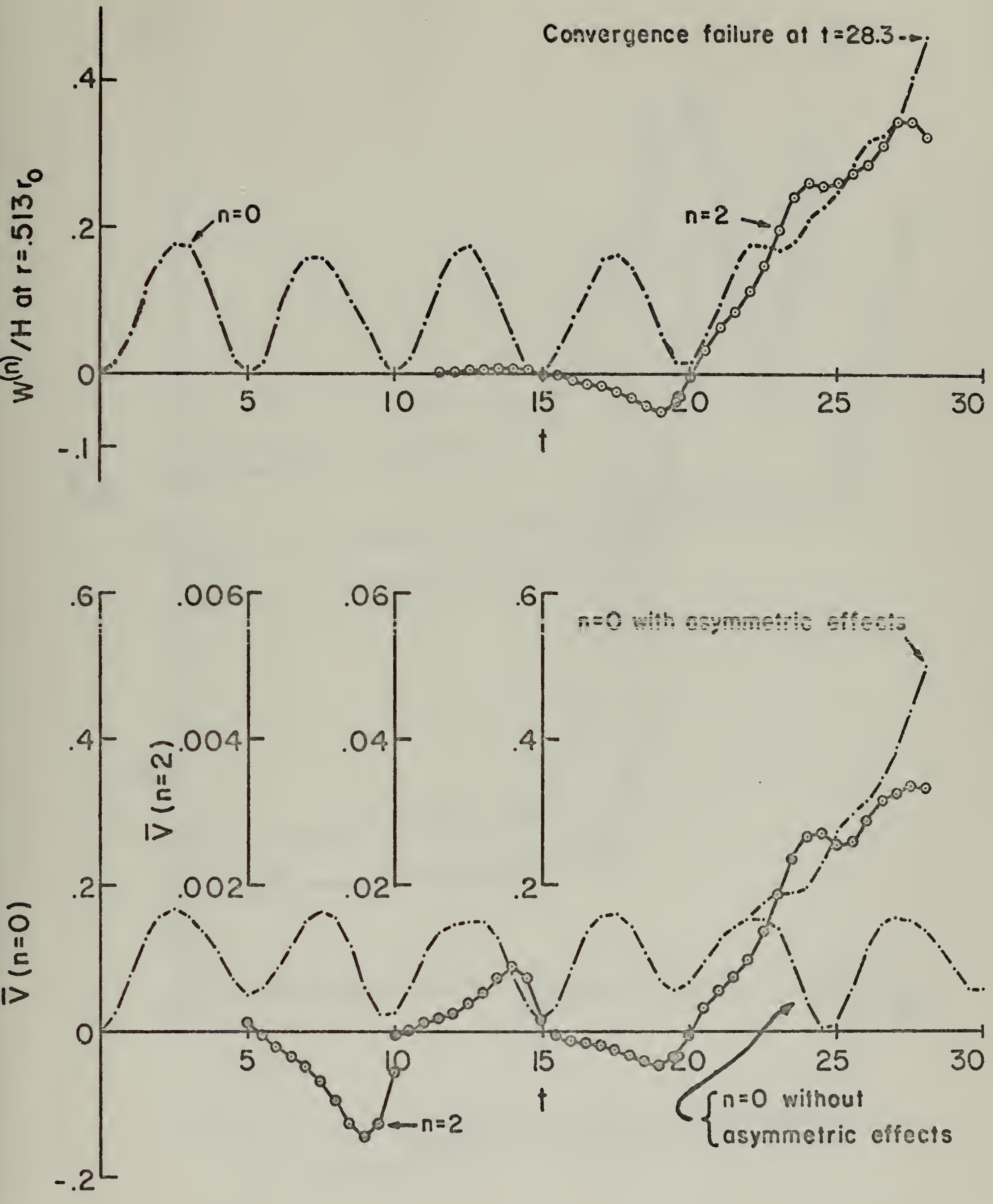

Figure 6. Normal deflection and volume deformation versus time for harmonics n = 0 and 2, $\lambda = 6$.

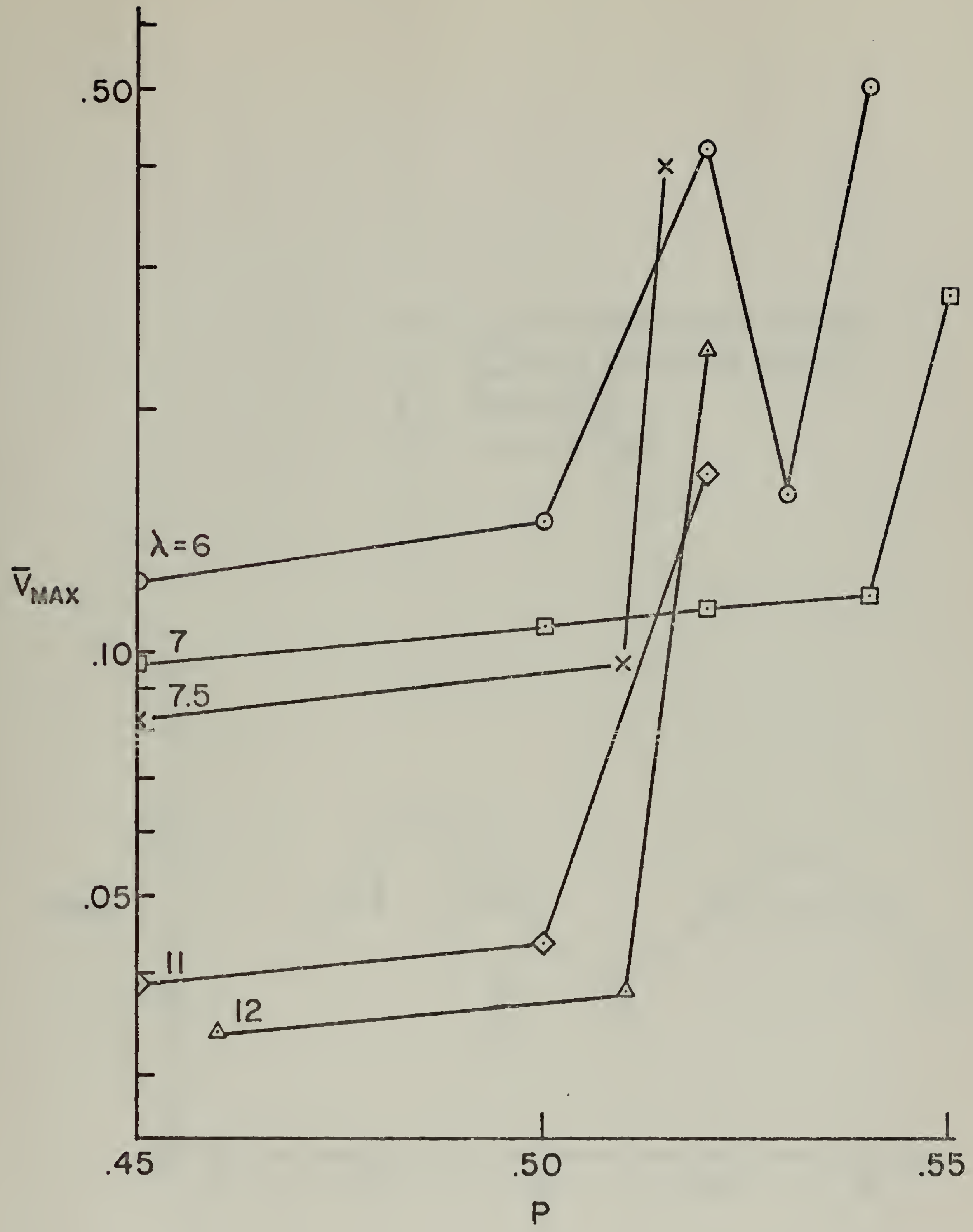

Figure 7. Peak deflection versus step pressure load from asymmetric analysis for various values of λ.

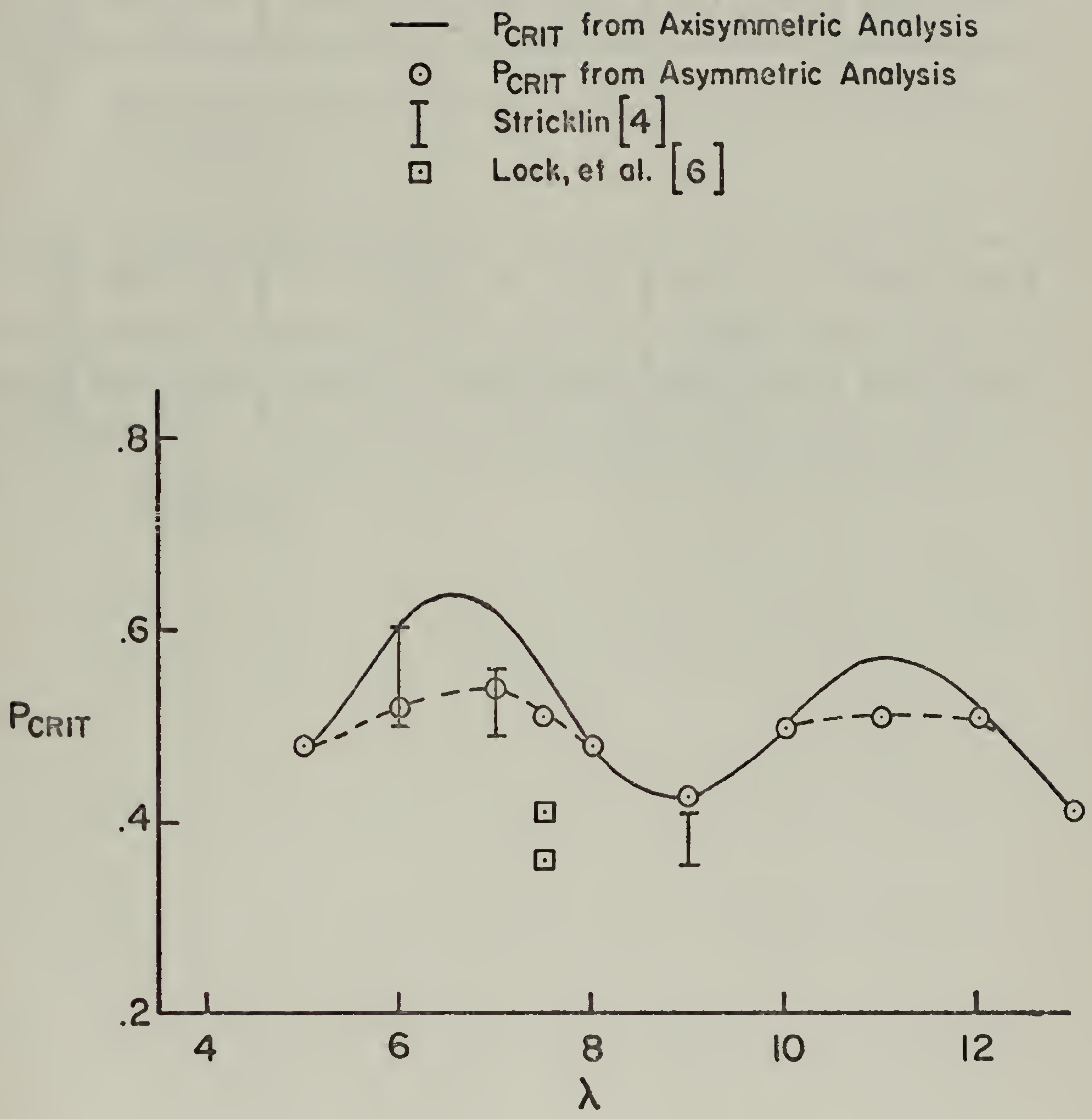

Figure 8. Critical step pressure load versus λ from asymmetric analysis.

λ	5	6	7	7.5	8	9	10	11	12	13
P_{CRIT}	.48	.61	.62	.54	.48	.43	.50	.575	.52	.41

Table I. Critical step pressure loads from axisymmetric analysis.

λ	5	6	7	7.5	8	9	10	11	12	13
P_{CRIT}	.48	.52	.54	.51	.48	.43	.50	.51	.51	.41

Table II. Critical step pressure loads from asymmetric analysis.

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REPORT TITLE
DYNAMIC BUCKLING OF SHALLOW SPHERICAL CAPS SUBJECTED TO A NEARLY AXISYMMETRIC STEP PRESSURE LOAD

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ABSTRACT

The behavior of shallow spherical caps subjected to axisymmetric and nearly axisymmetric step pressure loads is examined using a digital computer program for the geometrically nonlinear analysis of arbitrarily loaded shells of revolution. A criterion for dynamic buckling under the nearly axisymmetric load is proposed and critical buckling pressures are determined for both loading conditions for a large range of cap sizes. The axisymmetric results either compare well with, or are somewhat higher than, previously published theoretical results. For most of the cap sizes the buckling pressures from the asymmetric analysis are new results. A numerical convergence study is conducted to verify the accuracy of the results.

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